A simplified human birth model: translation of a rigid cylinder through a passive elastic tube

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Motivation

Vaginal delivery is linked to

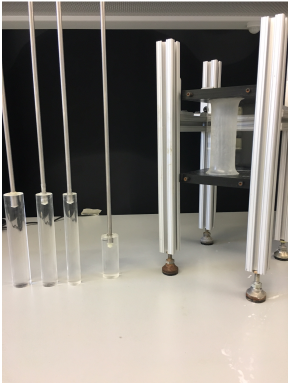
- shorter post-birth hospital stays
- lower likelihood of intensive care stays
- lower mortality rates [1]

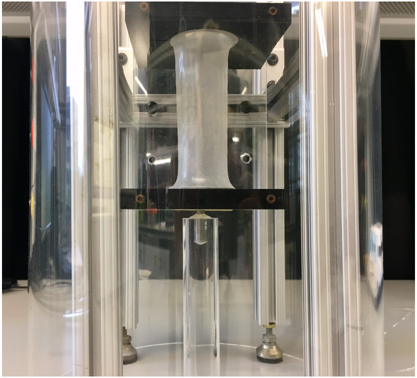
Fluid mechanics greatly informs the total mechanics of birth.

- vernix caseosa
- amniotic fluid

[1] C. S. Buhimschi, I. A. Buhimschi (2006). *Advantages of vaginal delivery*, Clinical obstetrics and gynecology.
Fig. 1: "HumanNewborn" by Ernest F - Own work.
Fig. 2: "Postpartum baby2" by Tom Adriaenssen - http://www.flickr.com/photos/inferis/110652572/.

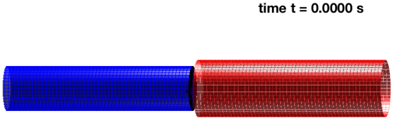
Physical Experiment





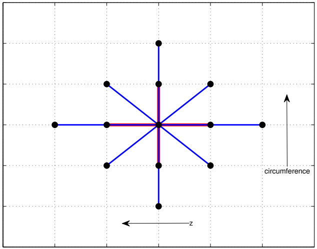
- Rigid acrylic cylinder (fetus)
- Passive elastic latex tube (birth canal)
- Viscous fluid - methyl cellulose and water (amniotic fluid)
- Rigid cylinder is pulled through center of elastic tube at constant velocity

Numerical Model



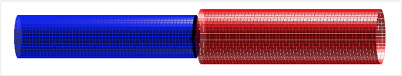
The Model: Solid Behavior

- Tube modeled by network of Hookean springs.
- Force at point $\mathbf{x}_l$ due to spring from point $\mathbf{x}_m$:
  $\mathbf{f}(\mathbf{x}_l) = \tau \left(\frac{\|\mathbf{x}_m - \mathbf{x}_l\|}{\Delta_{lm}} - 1 \right) \frac{(\mathbf{x}_m - \mathbf{x}_l)}{\|\mathbf{x}_m - \mathbf{x}_l\|}$
- τ chosen to match elastic properties to physical experiment. [2]



Rigid Inner Rod

- A constant velocity $\mathbf{u}$ is specified in the z-direction.



[2] H. Nguyen and L. Fauci (2014). *Hydrodynamics of diatom chains and semiflexible fibres*, J. R. Soc. Interface.

The Model: Fluid Dynamics

Fluid Behavior is governed by the Stokes equations, with regularized forces at K discrete points in the system:

$$0 = -\nabla p + \mu \Delta \mathbf{u} + \textstyle\sum_{k=0}^{K} \mathbf{f}_k \phi_\varepsilon(\mathbf{x} - \mathbf{x}_k), \nabla \cdot \mathbf{u} = 0,$$

which have solution [3],[4]

$$\mathbf{u}(\mathbf{x}) = \frac{1}{\mu} \sum_{k=1}^{K} \left[(\mathbf{f}_k \cdot \nabla) \nabla B_\varepsilon(|\mathbf{x} - \mathbf{x}_k|) - \mathbf{f}_k G_\varepsilon(|\mathbf{x} - \mathbf{x}_k|) \right],$$

$$p(\mathbf{x}) = \sum_{k=1}^{K} \left[\mathbf{f}_k \cdot \nabla G_\varepsilon(|\mathbf{x} - \mathbf{x}_k|) \right],$$

where $\Delta B_\varepsilon = G_\varepsilon, \Delta G_\varepsilon = \phi_\varepsilon(r) = \frac{15\varepsilon^4}{8\pi(r^2+\varepsilon^2)^{(7/2)}}$.

Here, μ is viscosity, $\mathbf{x}_k$ are points on discretized tube and rod, $\mathbf{f}_k$ is the force at that point, and ε is a regularization parameter.

[3] R. Cortez (2001). *Method of Regularized Stokeslets*, SIAM Journal of Scientific Computing.

[4] R. Cortez, L. Fauci, A. Medovikov (2005). *The method of regularized Stokeslets in three dimensions: analysis, validation, and application to helical swimming*, Physics of Fluids.

The Model: Algorithm

Using the solution to the regularized Stokes equations for a given regularization function ϕ_ε:

(1) Calculate spring forces in the tube based on its deformation, and calculate the velocity they induce on the inner cylinder.

(2) Solve for additional forces necessary on inner cylinder points to achieve its desired constant velocity.

(3) Evaluate the velocity points on tube and inner cylinder. (Velocity and pressure can be evaluated at any other point in the system.)

(4) Update the tube and rod positions using these velocities one step forward in time.

(5) Repeat.

Validation: Velocity Profile Between Cylinder and Solid Tube

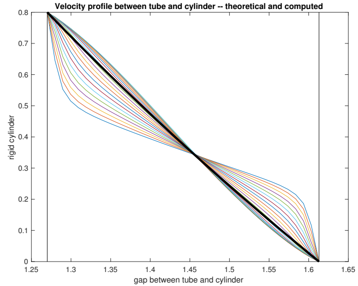
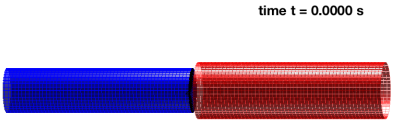
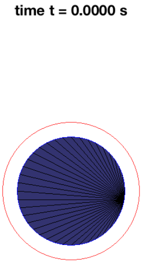


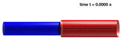
Figure: The velocity profile between an infinitely long rigid cylinder of radius R_c and an infinitely long rigid tube of radius R_t is $u(r) = \frac{U(\ln(R_t) - \ln(r))}{\ln(R_t) - \ln(R_c)}$. This is compared with the velocity computed using the method of regularized stokeslets with varying values for ε.

Results: System Behavior and Elastic Buckling

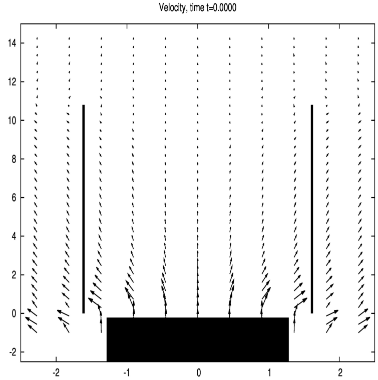


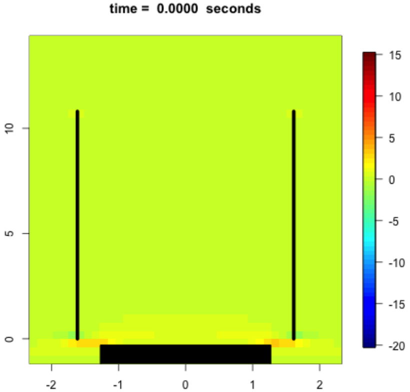
Results: System Behavior and Elastic Buckling



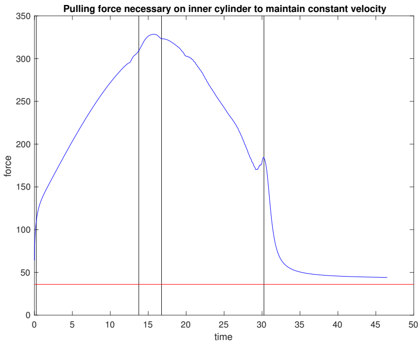


Results: Causes of Elastic Buckling





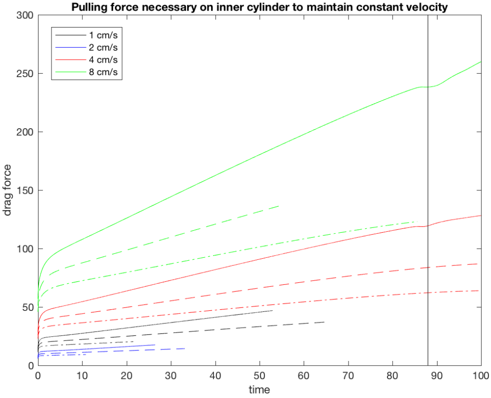
Results: Force on Rigid Cylinder



- In blue: Drag force on rigid inner cylinder as it moves at constant velocity through elastic tube
- In red: Asymptotic approximation to the drag force on a long, slender cylinder moving along its axis through a viscous fluid [5]: $\mathbf{F} = \frac{4\pi\mu UL}{2\ln(L/r)-1}$

[5] C. Pozrikidis (2011). *Introduction to theoretical and computational fluid dynamics*, Oxford University Press.

Results: Force Variation with Diameter and Velocity



- Force doubles when velocity doubles, as expected in Stokes flow regime
- Force decreases as diameter of inner cylinder decreases

Future Work

- Further analysis of tube buckling behavior
  - Variation of diameter and length of rigid cylinder and tube
  - Variation of velocity of rigid cylinder
  - Variation of elasticity of tube
- Increase realism
  - active elastic tube / modeling peristalsis
  - more accurate geometry

Slides available at
`math.tulane.edu/~rpealate`