



# **A simplified human birth model: translation of a rigid cylinder through a passive elastic tube**

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Work supported in part by NSF DMS 1043626

# Motivation

Vaginal delivery is linked to

- ▶ shorter post-birth hospital stays
- ▶ lower likelihood of intensive care stays
- ▶ lower mortality rates [1]

Fluid mechanics greatly informs the total mechanics of birth. [2]

- ▶ How do the involved fluids inform the forces on an infant during birth?



[1] C. S. Buhimschi, I. A. Buhimschi (2006). *Advantages of vaginal delivery*, Clinical obstetrics and gynecology.

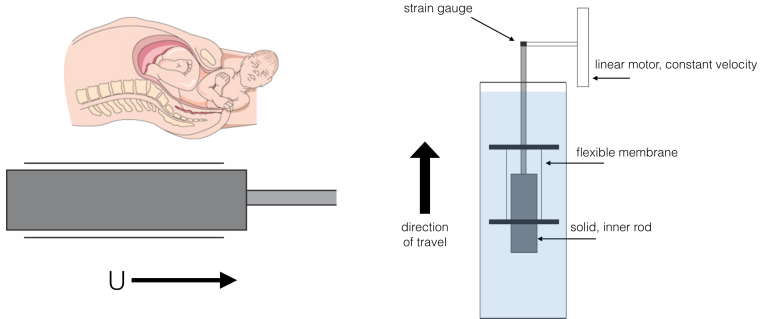
[2] A. M. Lehn, A. Baumer, M. C. Leftwich, *An experimental approach to a simplified model of human birth*.

Fig. 1: "HumanNewborn" by Ernest F - Own work. Licensed under CC BY-SA 3.0 via Commons - <https://commons.wikimedia.org/wiki/File:HumanNewborn.JPG#/media/File:HumanNewborn.JPG>

Fig. 2: "Postpartum baby2" by Tom Adriaenssen - <http://www.flickr.com/photos/inferis/110652572/>.

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# A simplified model



- ▶ Rigid acrylic cylinder (fetus)
- ▶ Passive elastic latex tube (birth canal)
- ▶ Viscous fluid - methyl cellulose and water (amniotic fluid)
- ▶ Rigid cylinder is pulled through center of elastic tube at constant velocity

# Numerical model: solid behavior

## Elastic tube

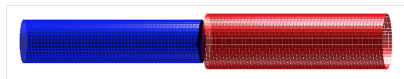
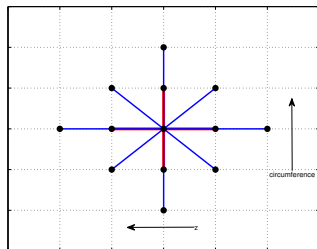
- ▶ Tube modeled by network of Hookean springs.
- ▶ Force at point  $\mathbf{x}_l$  due to spring from point  $\mathbf{x}_m$ :

$$\mathbf{g}(\mathbf{x}_l) = \tau \left( \frac{\|\mathbf{x}_m - \mathbf{x}_l\|}{\Delta_{lm}} - 1 \right) \frac{(\mathbf{x}_m - \mathbf{x}_l)}{\|\mathbf{x}_m - \mathbf{x}_l\|}$$

- ▶  $\tau$  chosen to match elastic properties to physical experiment. [3]

## Rigid inner cylinder

- ▶ A constant velocity  $\mathbf{u} = U$  is specified in the z-direction.



# Numerical model: fluid dynamics

**Fluid Behavior** is governed by the Stokes equations, with regularized forces at  $K$  discrete points in the system:

$$0 = -\nabla p + \mu \Delta \mathbf{u} + \sum_{k=0}^K \mathbf{f}_k \phi_\varepsilon(\mathbf{x} - \mathbf{x}_k), \nabla \cdot \mathbf{u} = 0,$$

which have solution [4],[5]

$$\mathbf{u}(\mathbf{x}) = \frac{1}{\mu} \sum_{k=1}^K [(\mathbf{f}_k \cdot \nabla) \nabla B_\varepsilon(|\mathbf{x} - \mathbf{x}_k|) - \mathbf{f}_k G_\varepsilon(|\mathbf{x} - \mathbf{x}_k|)],$$
$$p(\mathbf{x}) = \sum_{k=1}^K [\mathbf{f}_k \cdot \nabla G_\varepsilon(|\mathbf{x} - \mathbf{x}_k|)],$$

where  $\Delta B_\varepsilon = G_\varepsilon$ ,  $\Delta G_\varepsilon = \phi_\varepsilon(r) = \frac{15\varepsilon^4}{8\pi(r^2 + \varepsilon^2)^{(7/2)}}$ .

Here,  $\mu$  is viscosity,  $\mathbf{x}_k$  are points on discretized tube and rod,  $\mathbf{f}_k$  is the force at that point, and  $\varepsilon$  is a regularization parameter.

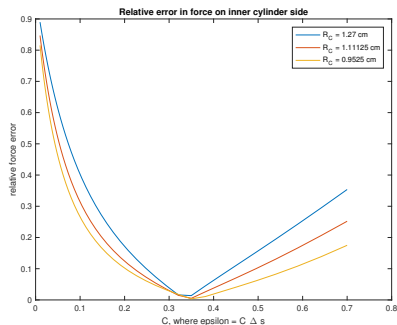
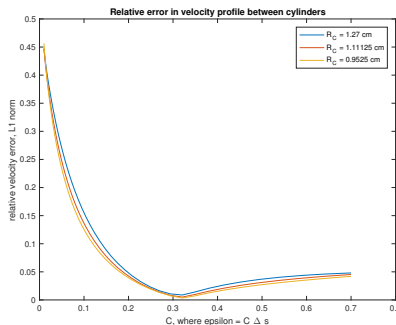
[4] R. Cortez (2001). *Method of Regularized Stokeslets*, SIAM Journal of Scientific Computing.

[5] R. Cortez, L. Fauci, A. Medovikov (2005). *The method of regularized Stokeslets in three dimensions: analysis, validation, and application to helical swimming*, Physics of Fluids.

# Validation: concentric rigid cylinders

For concentric rigid cylinders of infinite length, with outer tube of radius  $R_T$  fixed and inner cylinder of radius  $R_C$  moving at constant velocity  $U$ :

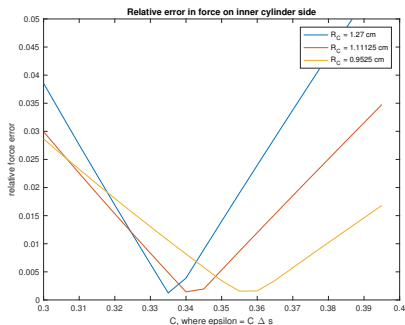
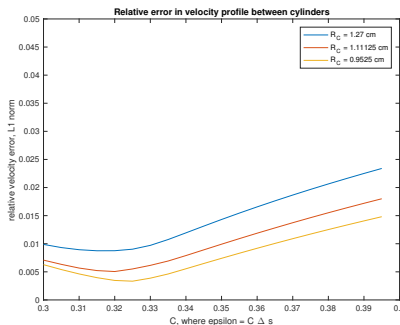
- ▶ Velocity profile between cylinders is given by:  $u(r) = \frac{U(\ln(R_T) - \ln(r))}{\ln(R_T) - \ln(R_C)}$
- ▶ Traction at a point on the side of inner cylinder is:  $t = \frac{\mu U}{R_C \ln\left(\frac{R_T}{R_C}\right)}$
- ▶ Compared to numerical results for finite-length concentric rigid cylinders:



# Validation: concentric rigid cylinders

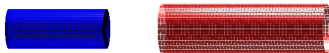
For concentric rigid cylinders of infinite length, with outer tube of radius  $R_T$  fixed and inner cylinder of radius  $R_C$  moving at constant velocity  $U$ :

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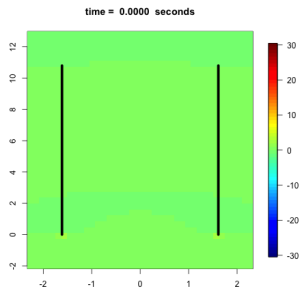
# Results: a sample simulation

$L=6.6\text{cm}$ ,  $R=1.27\text{cm}$ ,  $V=0.4\text{cm/s}$ ,  $t=0\text{s}$

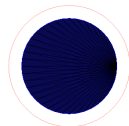




# Results: a sample simulation

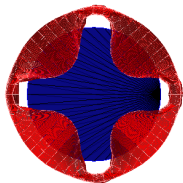


$L=6.6\text{cm}$ ,  $R=1.27\text{cm}$ ,  $V=0.4\text{cm/s}$ ,  $t=0\text{s}$

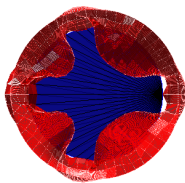


# Results: tube buckling

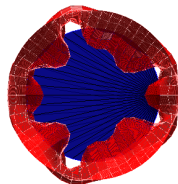
$U = 0.4$



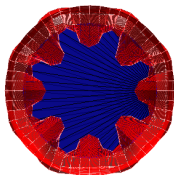
$U = 0.8$



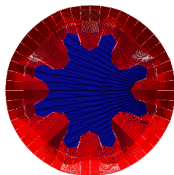
$U = 1.6$



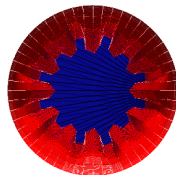
$U = 3.2$



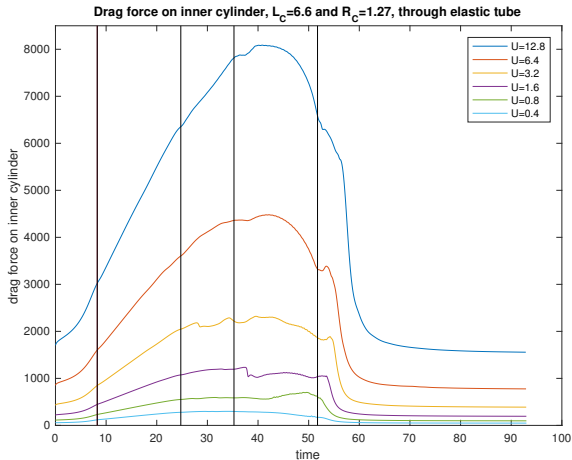
$U = 6.4$



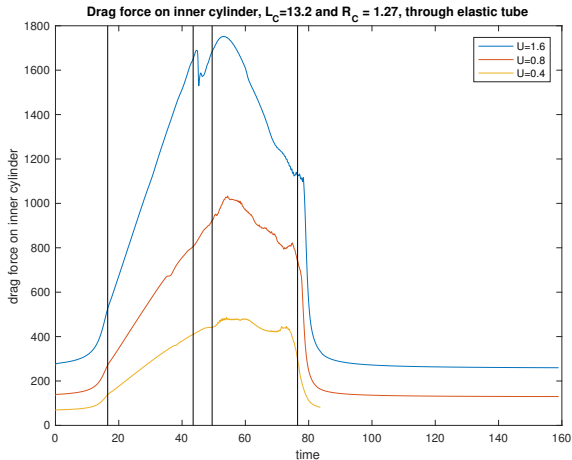
$U = 12.8$



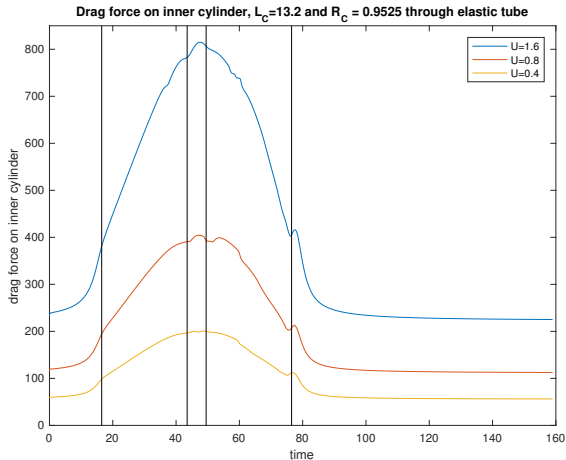
# Results: force on rigid inner cylinder



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## Finding dimensionless elastohydrodynamic numbers

The dimensionless ratio of fluid and elastic “forces”

$$\frac{\mu UL^3}{\mathcal{E}I}$$

arises from standard dimensional analysis methods, where

$\mu$  = fluid viscosity,

$U$  = fluid velocity,

$\mathcal{E}$  = Young's modulus of tube,

$I$  = second moment of area of tube,

$L$  could be taken to be the tube's length or radius, or the inner cylinder's length or radius.

Keeping the geometry constant, this is still meaningful.

# Tube buckling under constant elastohydrodynamic number

$L=3.3\text{cm}$ ,  $R=0.9525\text{cm}$ ,  $V=0.2\text{cm/s}$ ,  $\mu=2.0$ ,  $E=0.5 \cdot 10^5$ ,  $t=0\text{s}$

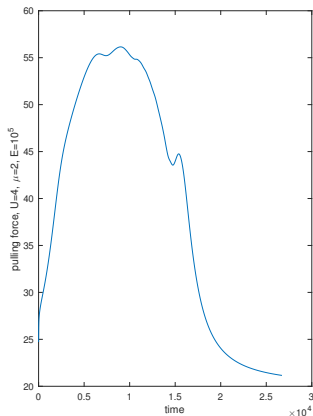
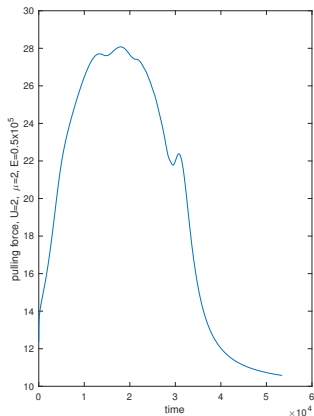


$L=3.3\text{cm}$ ,  $R=0.9525\text{cm}$ ,  $V=0.4\text{cm/s}$ ,  $\mu=2.0$ ,  $E=10^5$ ,  $t=0\text{s}$



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# Pulling force under constant elastohydrodynamic number





# Future work

- ▶ Further analysis of tube buckling behavior
  - ▶ How does the relationship between inner cylinder velocity and tube buckling behavior change with the dimensions of the inner cylinder?
  - ▶ With variation of elasticity of the tube?
  - ▶ Is it possible to find a dimensionless parameter associated with invariant buckling behavior that does not require a constant geometry?
- ▶ Increase realism
  - ▶ active elastic tube / modeling peristalsis
  - ▶ more accurate geometry

Slides available at  
[math.tulane.edu/~rpealate](http://math.tulane.edu/~rpealate)

