A numerical investigation of a simplified human birth model

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Motivation

Vaginal delivery is linked to

- shorter post-birth hospital stays
- lower likelihood of intensive care stays
- lower mortality rates [1]

Fluid mechanics greatly informs the total mechanics of birth.

- vernix caseosa
- amniotic fluid

[1] A. M. Lehn, A. Baumer and M. C. Leftwich (2015). *An experimental approach to a simplified model of human birth.*

Fig. 1: "HumanNewborn" by Ernest F - Own work. Licensed under CC BY-SA 3.0 via Commons - https://commons.wikimedia.org/wiki/File:HumanNewborn.JPG#/media/File:HumanNewborn.JPG

Fig. 2: "Postpartum baby2" by Tom Adriaenssen - http://www.flickr.com/photos/inferis/110652572/. Licensed under CC BY-SA 2.0 via Commons - https://commons.wikimedia.org/wiki/File:Postpartum_baby2.jpg#/media/File:Postpartum_baby2.jpg

Physical Experiment

- birth canal modeled by elastic latex tube
- fetus modeled by solid glass cylinder
- amniotic fluid modeled by viscous fluid (water/methyl cellulose mixture)

The Model: Solid Behavior

Elastic Tube

- Tube modeled by network of Hookean springs.
- Spring force acting on each point by the spring connecting it to one other point is
$$\mathbf{f}(\mathbf{x}_l) = \tau \left(\frac{||\mathbf{x}_m - \mathbf{x}_l||}{\Delta_{lm}} - 1 \right) \frac{(\mathbf{x}_m - \mathbf{x}_l)}{||\mathbf{x}_m - \mathbf{x}_l||}$$
- Elastic properties specified to match physical experiment by selection of spring constant τ. [2]

Rigid Inner Rod

- A constant velocity $\mathbf{u}$ is specified in the z-direction.

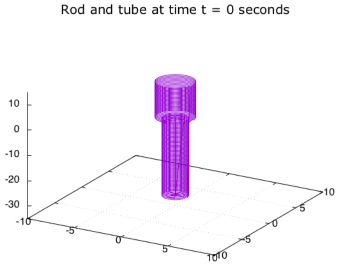


Figure : Discretization of rod and tube position in fluid at beginning of simulation.

[2] H. Nguyen and L. Fauci (2014). *Hydrodynamics of diatom chains and semiflexible fibres*, J. R. Soc. Interface.

The Model: Fluid Dynamics

Fluid Behavior

- Low Reynolds number $\implies$ fluid behavior governed by Stokes equations: $0 = -\nabla p + \mu \Delta \mathbf{u} + \mathbf{f}, \nabla \cdot \mathbf{u} = 0.$
- Given a single point force, we can find the solution to $0 = -\nabla p + \mu \Delta \mathbf{u} + \mathbf{f}_0 \delta(\mathbf{x} - \mathbf{x}_0)$, $\nabla \cdot \mathbf{u} = 0.$
- This can be regularized by approximating the dirac-δ function with a blob function ϕ_ε. [3]

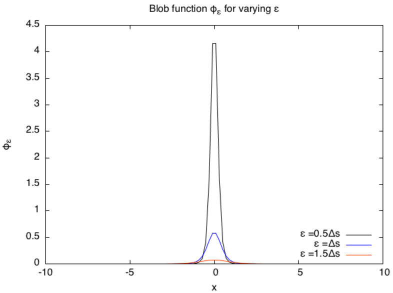


Figure : Various blob functions ϕ_ε to approximate $\delta(x)$.

[3] R. Cortez (2001). *Method of Regularized Stokeslets*, SIAM Journal of Scientific Computing.

The Model: Fluid Dynamics

In three dimensions, using the blob function

$$\phi_\varepsilon(r) = \frac{15\varepsilon^4}{8\pi(r^2+\varepsilon^2)^{(7/2)}},$$

we can express the linear relationship between fluid velocities and regularized forces that are localized at N points [4]:

$$\mathbf{u}(\mathbf{x}) = \frac{1}{\mu}\sum_{k=1}^{N}\left(\mathbf{f}_k\left[\frac{r_k^2+2\varepsilon^2}{8\pi(r_k^2+\varepsilon^2)^{(3/2)}}\right] + \left[\mathbf{f}_k\cdot((\mathbf{x}-\mathbf{x}_k)\right](\mathbf{x}-\mathbf{x}_k)\left[\frac{1}{8\pi(r_k^2+\varepsilon^2)^{(3/2)}}\right]\right),$$
$$p(\mathbf{x}) = \sum_{k=1}^{N}\left(\left[\mathbf{f}_k\cdot(\mathbf{x}-\mathbf{x}_k)\right]\frac{2r_k^2+5\varepsilon^2}{8\pi(r_k^2+\varepsilon^2)^{(5/2)}}\right),$$

where μ is viscosity, $\mathbf{x}_k$ are points on discretized tube and rod, and $\mathbf{f}_k$ is the force at that point. Here, ε is a regularization parameter.

[4] R. Cortez, L. Fauci, A. Medovikov (2005). *The method of regularized Stokeslets in three dimensions: analysis, validation, and application to helical swimming*, Physics of Fluids.

The Model: Numerical Solution

Using the solution to the regularized Stokes equations for a given blob function, we can

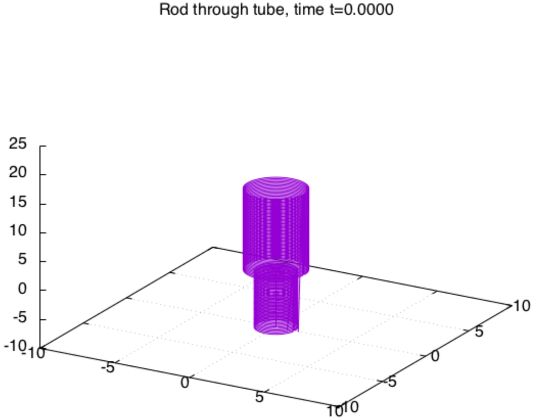
(1) find the velocity induced on the rod by spring forces in the tube,

(2) solve for any additional forces on the rod necessary to achieve its prescribed velocity,

(3) evaluate the velocity and pressure at every point in the system,

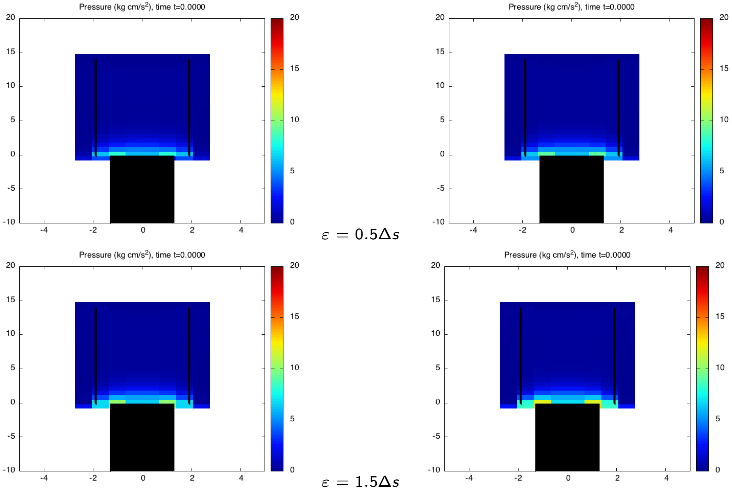
(4) update the tube and rod positions using these velocities one step forward in time.

Results: System Behavior



$\varepsilon = 1.5\Delta s$

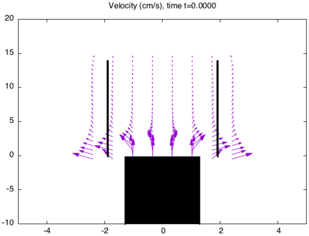
Results: Fluid Pressure



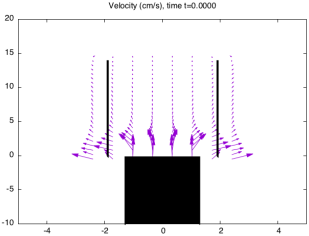
$\varepsilon = 0.5\Delta s$ $\varepsilon = \Delta s$

$\varepsilon = 1.5\Delta s$ $\varepsilon = 2\Delta s$

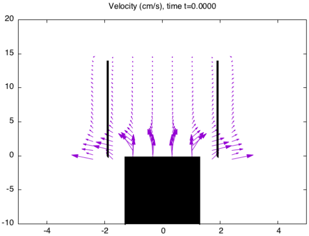
Results: Fluid Velocity



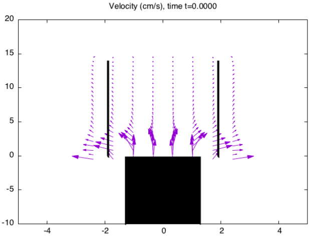
$\varepsilon = 0.5\Delta s$



$\varepsilon = \Delta s$



$\varepsilon = 1.5\Delta s$



$\varepsilon = 2\Delta s$

Results: Pulling Force

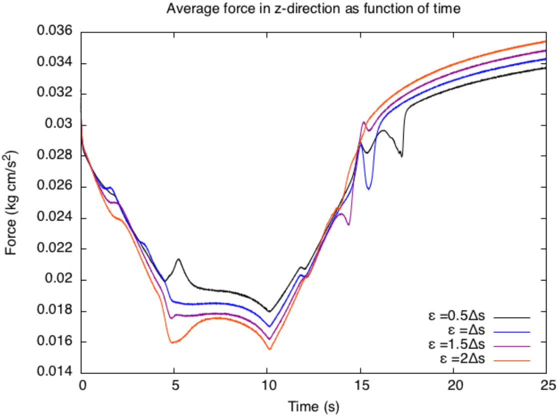


Figure : Necessary pulling force to move rod through tube at prescribed constant velocity, plotted against time.

Future Work

- Matching physical experiment
  - finer spatial discretization
  - choice of blob parameter ε
- Alternative elastic models
  - continuum model of elastic tube
- Increasing realism
  - more accurate dimensions
  - active elastic tube / modeling peristalsis
  - more accurate geometry

Questions?