A numerical investigation of a simplified human birth model

Roseanna Pealatere[1], Alexa Baumer[2], Lisa Fauci[1], Megan C. Leftwich[2]
Tulane University[1], The George Washington University[2]
Work supported in part by NSF DMS 1043626

Motivation

Vaginal delivery is linked to
- shorter post-birth hospital stays
- lower likelihood of intensive care stays
- lower mortality rates [1]

Fluid mechanics greatly informs the total mechanics of birth. [4]

Figure 1.

Problem Description

To understand how amniotic fluid informs the forces on the fetus during birth, a simple physical model was constructed: a rigid cylinder (fetus) passes through the center of a passive elastic tube (birth canal). The system is immersed in water mixed with methyl cellulose (amniotic fluid). Our aim is to calculate the forces on the rigid cylinder and track the evolution of the geometry of the tube.

Figure 2. Physical experiment set-up at Leftwich Laboratory[2]

Mathematical Background

Much work has been done studying fluid flow through elastic tubes with fixed ends in three dimensions. [3]
- In previous models, tube dynamics have been modeled using nonlinear shell theory and viscous fluid dynamics using lubrication theory.
- Non-axisymmetric tube collapse occurs when the transmural pressure reaches a critically low value.

Numerical Methods

Elastic Tube
- Tube modeled by network of Hookean springs.
- Force at $\mathbf{x}_l$ due to spring from $\mathbf{x}_m$: $\mathbf{g}(\mathbf{x}_l) = \tau \left(\frac{\|\mathbf{x}_m - \mathbf{x}_l\|}{\Delta_{lm}} - 1 \right) \frac{(\mathbf{x}_m - \mathbf{x}_l)}{\|\mathbf{x}_m - \mathbf{x}_l\|}$
- τ chosen to match elastic properties to physical experiment. [5]

Rigid Inner Rod
- A constant velocity $\mathbf{u}$ is specified in the z-direction.

Fluid governed by the Stokes equations:

$$0 = -\nabla p + \mu \Delta \mathbf{u} + \Sigma_{k=1}^{N} \mathbf{f}_k,$$
$$\nabla \cdot \mathbf{u} = 0,$$

where $\mathbf{f}_k$ is the total force on the point $\mathbf{x}_k$.

The linear relationship between fluid velocities and regularized forces localized at N points is given by

$$\mathbf{u}(\mathbf{x}) = \frac{1}{\mu} \sum_{k=1}^{N} \left[(\mathbf{f}_k \cdot \nabla) \nabla B_\varepsilon(|\mathbf{x} - \mathbf{x}_k|) - \mathbf{f}_k G_\varepsilon(|\mathbf{x} - \mathbf{x}_k|) + \mathbf{u}_b(\mathbf{x}) \right],$$
$$p(\mathbf{x}) = \sum_{k=1}^{N} \left[\mathbf{f}_k \cdot \nabla G_\varepsilon(|\mathbf{x} - \mathbf{x}_k|) \right],$$

where $\Delta B_\varepsilon = G_\varepsilon, \Delta G_\varepsilon = \phi_\varepsilon, \phi_\varepsilon(r) = \frac{15\varepsilon^4}{8\pi(r^2+\varepsilon^2)^{(7/2)}}$, μ =viscosity, ε regularization parameter. [2]

Algorithm

(1) find the velocity induced on the rigid inner cylinder by spring forces in the tube,
(2) solve for any additional forces on the inner cylinder necessary to achieve its prescribed velocity,
(3) evaluate the velocity and pressure at every point in the system,
(4) update the tube and cylinder positions using these velocities one step forward in time.

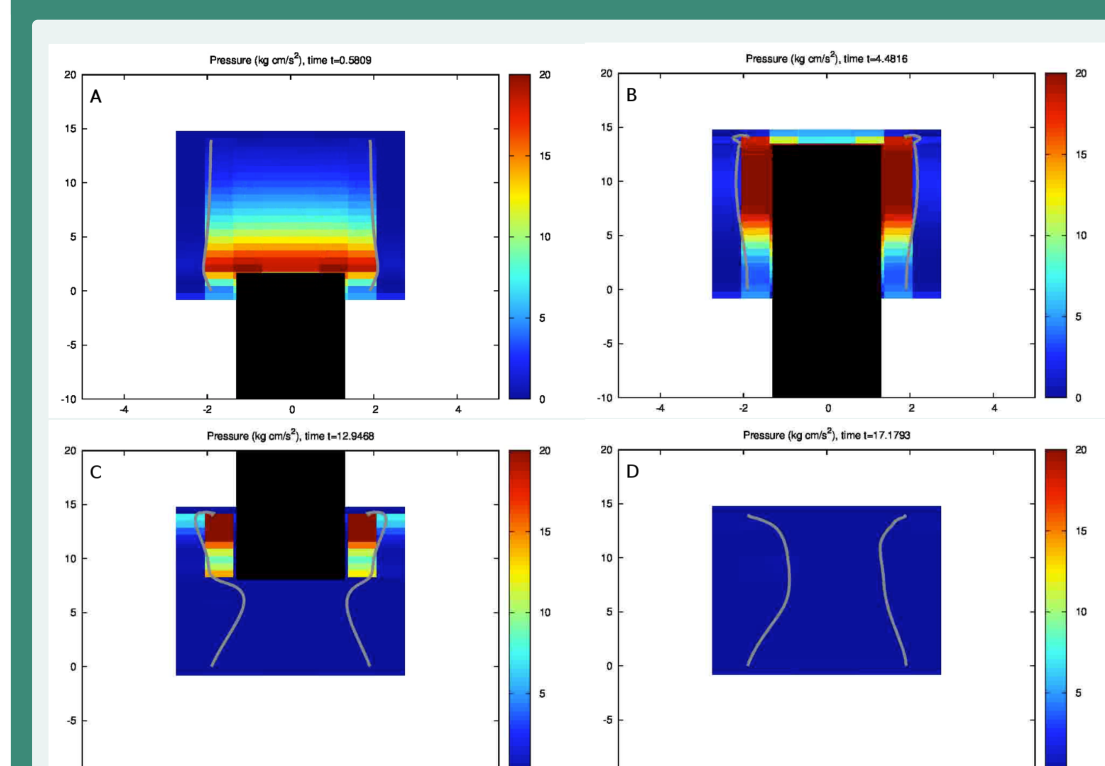

Figure 3. Elastic tube, rigid inner cylinder, and fluid pressure in cross section. As the inner cylinder exits the tube, the pressure drops and the tube buckles.

Figure 4. The rigid inner cylinder (blue) and elastic tube (red) shown throughout simulation.

Tube Buckling

Figure 5. Tube (viewed from end) deforms with circumferential wave number 6 after buckling due to pressure drop.

Discussion

As the inner cylinder enters the elastic tube, fluid (and so the tube) is pushed outward. As the pressure drops immediately behind the rigid cylinder as it moves, fluid rushes inward, causing non-axisymmetric tube buckling. The buckling wave number can be shown to be affected by system dimensions, tube elasticity, and inner cylinder speed.

Future Work

- Determine causes of specific buckling behavior.
- Use a continuum elastic model for the tube and compare system behavior.
- Increase realism with better geometry and/or active peristalsis in tube.

References

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[4] A. M. Lehn, A. Baumer, M. C. Leftwich, *An experimental approach to a simplified model of human birth*, preprint (2015).

[5] H. Nguyen and L. Fauci, *Hydrodynamics of diatom chains and semiflexible fibres*, J. R. Soc. Interface 11: 20140314 (2014).

[6] Fig.1: "HumanNewborn" by Ernest F - Own work. Licensed under CC BY-SA 3.0 via Commons - https://commons.wikimedia.org/wiki/File:HumanNewborn.JPG#/media/File:HumanNewborn.JPG, "Postpartum baby2" by Tom Adriaenssen - http://www.flickr.com/photos/inferis/110652572/. Licensed under CC BY-SA 2.0 via Commons - https://commons.wikimedia.org/wiki/File:Postpartum_baby2.jpg#/media/File:Postpartum_baby2.jpg