A simplified human birth model: translation of a rigid cylinder through a passive elastic tube

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Work supported in part by NSF DMS 1043626

Motivation

Vaginal delivery is linked to

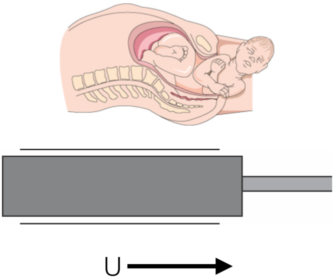
- shorter post-birth hospital stays
- lower likelihood of intensive care stays
- lower mortality rates [1]

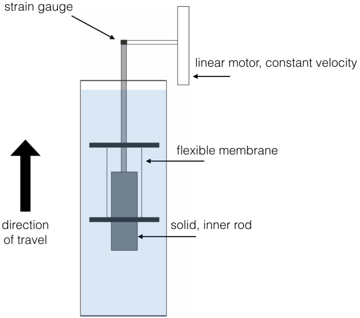
Fluid mechanics greatly informs the total mechanics of birth. [2]

- How do the involved fluids inform the forces on an infant during birth?

[1] C. S. Buhimschi, I. A. Buhimschi (2006). *Advantages of vaginal delivery*, Clinical obstetrics and gynecology.
[2] A. M. Lehn, A. Baumer, M. C. Leftwich, *An experimental approach to a simplified model of human birth*.

A simplified model



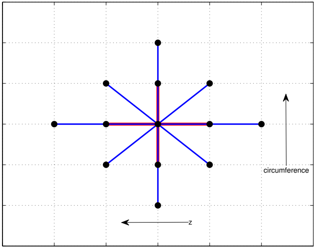


- Rigid acrylic cylinder (fetus)
- Passive elastic latex tube (birth canal)
- Viscous fluid - methyl cellulose and water (amniotic fluid)
- Rigid cylinder is pulled through center of elastic tube at constant velocity

Numerical model: solid behavior

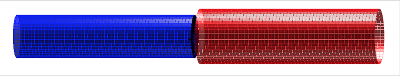
Elastic tube

- Tube modeled by network of Hookean springs.
- Force at point $\mathbf{x}_l$ due to spring from point $\mathbf{x}_m$:
  $\mathbf{g}(\mathbf{x}_l) = \tau \left(\frac{\|\mathbf{x}_m - \mathbf{x}_l\|}{\Delta_{lm}} - 1 \right) \frac{(\mathbf{x}_m - \mathbf{x}_l)}{\|\mathbf{x}_m - \mathbf{x}_l\|}$
- τ chosen to match elastic properties to physical experiment. [3]



Rigid inner cylinder

- A constant velocity $\mathbf{u} = U$ is specified in the z-direction.



[3] H. Nguyen and L. Fauci (2014). *Hydrodynamics of diatom chains and semiflexible fibres*, J. R. Soc. Interface.

Numerical model: fluid dynamics

Fluid Behavior is governed by the Stokes equations, with regularized forces at K discrete points in the system:

$$0 = -\nabla p + \mu \Delta \mathbf{u} + \textstyle\sum_{k=0}^{K} \mathbf{f}_k \phi_\varepsilon(\mathbf{x} - \mathbf{x}_k), \nabla \cdot \mathbf{u} = 0,$$

which have solution [4],[5]

$$\mathbf{u}(\mathbf{x}) = \frac{1}{\mu} \sum_{k=1}^{K} \left[(\mathbf{f}_k \cdot \nabla) \nabla B_\varepsilon(|\mathbf{x} - \mathbf{x}_k|) - \mathbf{f}_k G_\varepsilon(|\mathbf{x} - \mathbf{x}_k|) \right],$$

$$p(\mathbf{x}) = \sum_{k=1}^{K} \left[\mathbf{f}_k \cdot \nabla G_\varepsilon(|\mathbf{x} - \mathbf{x}_k|) \right],$$

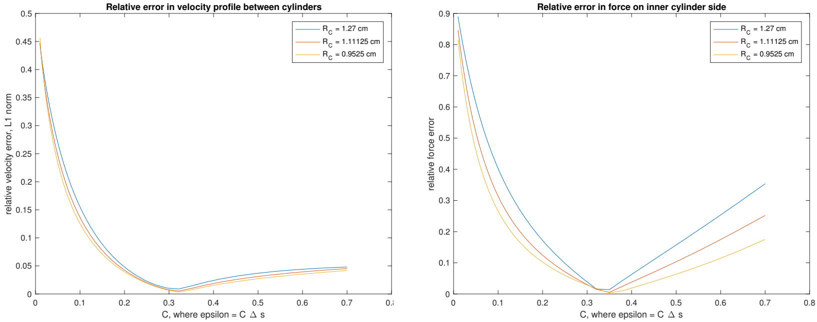
where $\Delta B_\varepsilon = G_\varepsilon, \Delta G_\varepsilon = \phi_\varepsilon(r) = \frac{15\varepsilon^4}{8\pi(r^2+\varepsilon^2)^{(7/2)}}$.
Here, μ is viscosity, $\mathbf{x}_k$ are points on discretized tube and rod, $\mathbf{f}_k$ is the force at that point, and ε is a regularization parameter.

[4] R. Cortez (2001). *Method of Regularized Stokeslets*, SIAM Journal of Scientific Computing.
[5] R. Cortez, L. Fauci, A. Medovikov (2005). *The method of regularized Stokeslets in three dimensions: analysis, validation, and application to helical swimming*, Physics of Fluids.

Validation: concentric rigid cylinders

For concentric rigid cylinders of infinite length, with outer tube of radius R_T fixed and inner cylinder of radius R_C moving at constant velocity U:

- Velocity profile between cylinders is given by: $u(r) = \frac{U(\ln(R_t) - \ln(r))}{\ln(R_t) - \ln(R_c)}$
- Traction at a point on the side of inner cylinder is: $t = \frac{\mu U}{R_c \ln\left(\frac{R_T}{R_C}\right)}$
- Compared to numerical results for finite-length concentric rigid cylinders:



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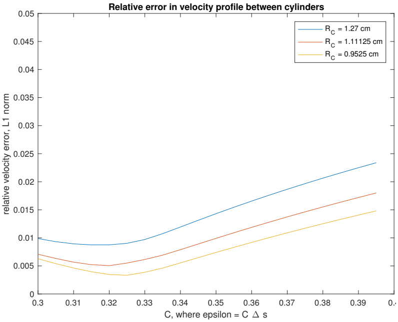
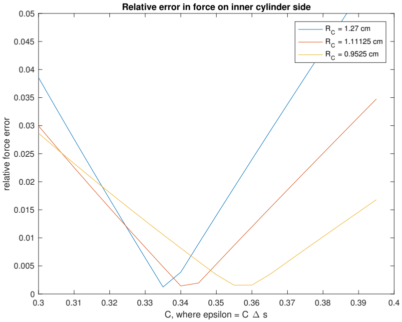
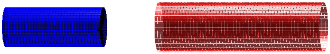




Results: a sample simulation

L=6.6cm, R=1.27cm, V=0.4cm/s, t=0s



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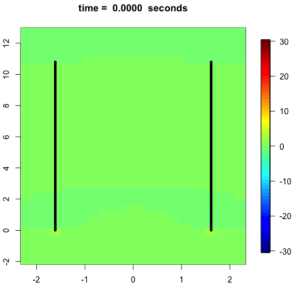
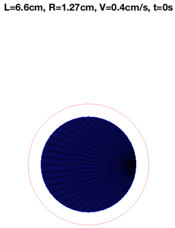
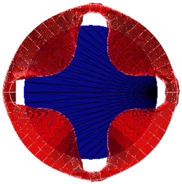


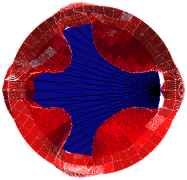


Results: tube buckling

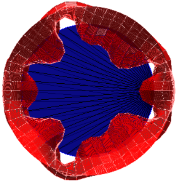
$U = 0.4$



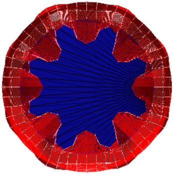
$U = 0.8$



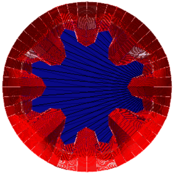
$U = 1.6$



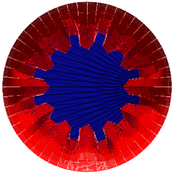
$U = 3.2$



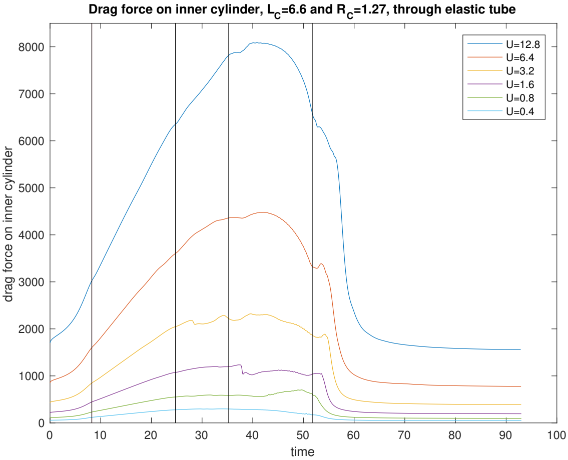
$U = 6.4$



$U = 12.8$



Results: force on rigid inner cylinder



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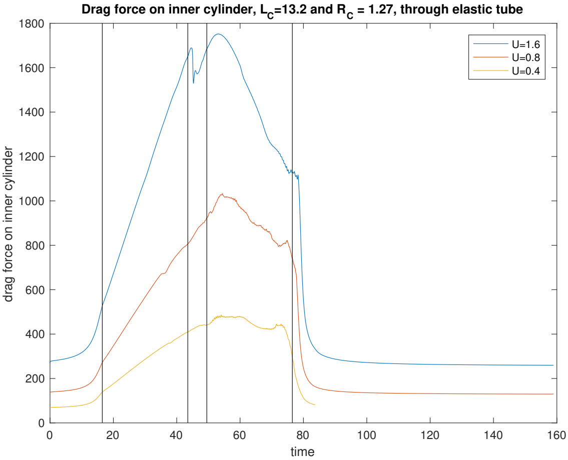

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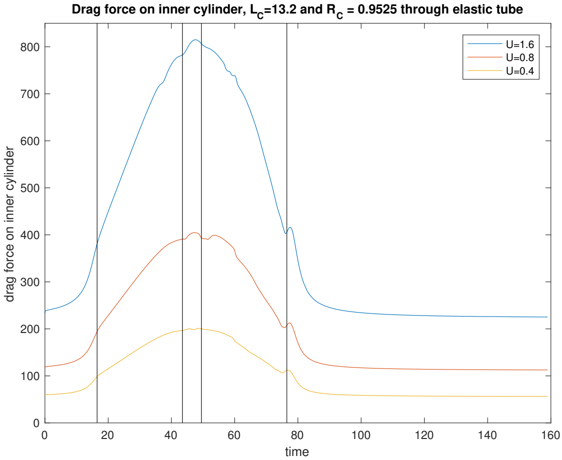

Future work

- Further analysis of tube buckling behavior
  - How does the relationship between inner cylinder velocity and tube buckling behavior change with the dimensions of the inner cylinder?
  - With variation of elasticity of the tube?
- Increase realism
  - active elastic tube / modeling peristalsis
  - more accurate geometry

Slides available at
`math.tulane.edu/~rpealate`