# A numerical investigation of a simplified human birth model

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## Motivation

When compared with Caesarean delivery, vaginal delivery is linked to

- shorter post-birth hospital stays
- lower likelihood of intensive care stays
- lower mortality rates [1]

Greater understanding of the causes of force on the infant during childbirth could decrease the occurrence of unnecessary Caesarean deliveries.

Fluid mechanics greatly informs the total mechanics of birth. [4] We aim to discover how the involved fluids affect forces on the infant during birth.

*Figure 1.*

## Problem Description

To understand how amniotic fluid informs the forces on the fetus during birth, a simple physical model was constructed: a rigid cylinder (fetus) passes through the center of a passive elastic tube (birth canal). The system is immersed in water mixed with methyl cellulose (amniotic fluid). Our aim is to calculate the forces on the rigid cylinder and track the evolution of the geometry of the tube.

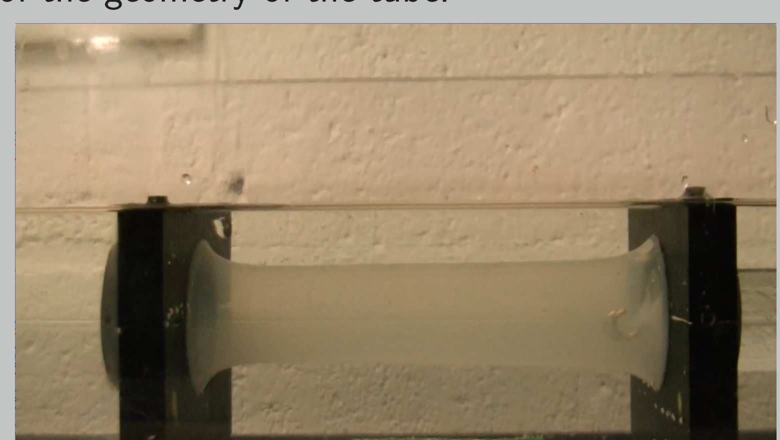

*Figure 2.* Physical experiment set-up at Leftwich Laboratory[2]

In the following numerical experiment, matching the physical experiment, we pull a 13.2cm long rigid rod with diameter 2.54cm through the center of a 10.8cm long elastic tube with diameter 3.23cm at a constant velocity of 0.8cm/s.

## Mathematical Background

Much work has been done studying fluid flow through elastic tubes with fixed ends in three dimensions. [3]

- In previous models, tube dynamics have been modeled using nonlinear shell theory and viscous fluid dynamics using lubrication theory.
- Non-axisymmetric tube collapse occurs when the transmural pressure reaches a critically low value.

## Numerical Methods

**Elastic Tube**

- Tube modeled by network of Hookean springs.
- Force at $\mathbf{x}_l$ due to spring from $\mathbf{x}_m$: $\mathbf{g}(\mathbf{x}_l) = \tau \left( \frac{\|\mathbf{x}_m - \mathbf{x}_l\|}{\Delta_{lm}} - 1 \right) \frac{(\mathbf{x}_m - \mathbf{x}_l)}{\|\mathbf{x}_m - \mathbf{x}_l\|}$
- $\tau$ chosen to match elastic properties to physical experiment. [5]

**Rigid Inner Rod**

- A constant velocity $\mathbf{u}$ is specified in the $z$-direction.

**Fluid** governed by the Stokes equations:

$$0 = -\nabla p + \mu \Delta \mathbf{u} + \textstyle\sum_{k=1}^{N} \mathbf{f}_k, \qquad \nabla \cdot \mathbf{u} = 0,$$

where $\mathbf{f}_k$ is the total force on the point $\mathbf{x}_k$.

The linear relationship between fluid velocities and regularized forces localized at $N$ points is given by

$$\mathbf{u}(\mathbf{x}) = \frac{1}{\mu} \sum_{k=1}^{N} \left[ (\mathbf{f}_k \cdot \nabla) \nabla B_\varepsilon(|\mathbf{x} - \mathbf{x}_k|) - \mathbf{f}_k G_\varepsilon(|\mathbf{x} - \mathbf{x}_k|) + \mathbf{u}_b(\mathbf{x}) \right],$$

$$p(\mathbf{x}) = \sum_{k=1}^{N} \left[ \mathbf{f}_k \cdot \nabla G_\varepsilon(|\mathbf{x} - \mathbf{x}_k|) \right],$$

where $\Delta B_\varepsilon = G_\varepsilon, \Delta G_\varepsilon = \phi_\varepsilon, \phi_\varepsilon(r) = \frac{15\varepsilon^4}{8\pi(r^2+\varepsilon^2)^{(7/2)}}$, $\mu$ =viscosity, $\varepsilon$ regularization parameter. [2]

## Algorithm

1. find the velocity induced on the rod by spring forces in the tube,
2. solve for any additional forces on the rod necessary to achieve its prescribed velocity,
3. evaluate the velocity and pressure at every point in the system,
4. update the tube and rod positions using these velocities one step forward in time.

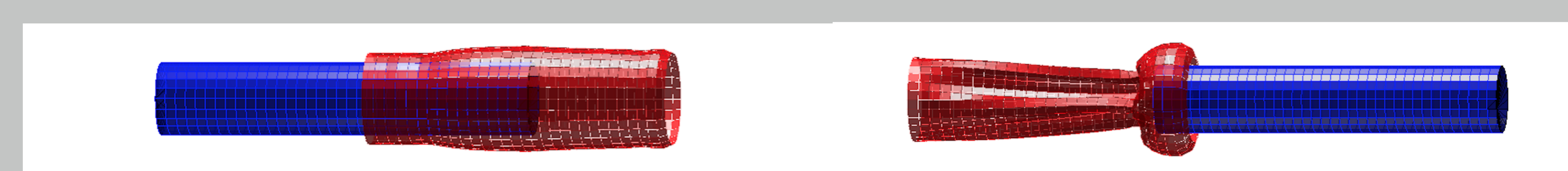

*Figure 3.* The rod (blue) and tube (red) at times $t \approx 15, 30$ s.

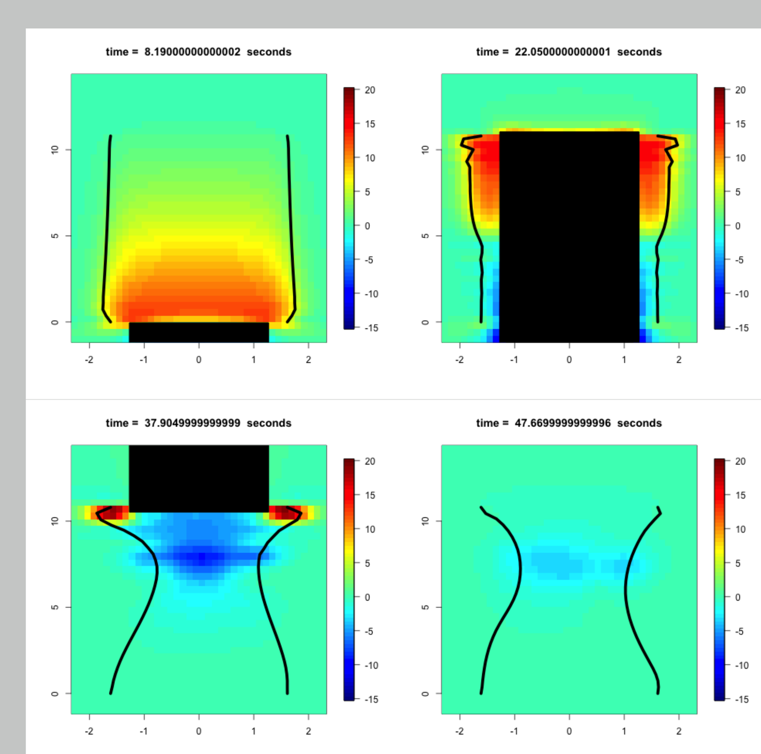

*Figure 4.* Tube, rod, and fluid pressure in cross section. As the rod exits the tube, the pressure drops and the tube buckles. Buckling reduces as the pressure inside the tube rises again.

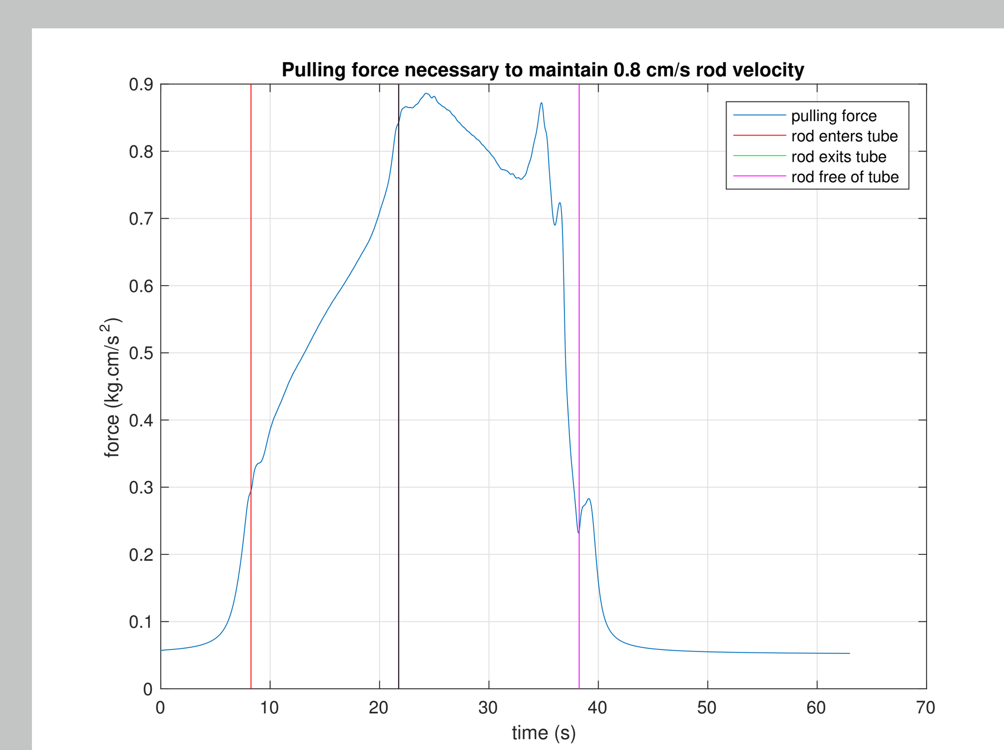


*Figure 5.* Rod is entering tube until $\approx$ **8**s, begins exiting the tube at $\approx$ **21**s, and is completely out of the tube at $\approx$ **38**s, after which the pulling force approaches a constant value as the rod interacts less with the tube.

## Tube Buckling

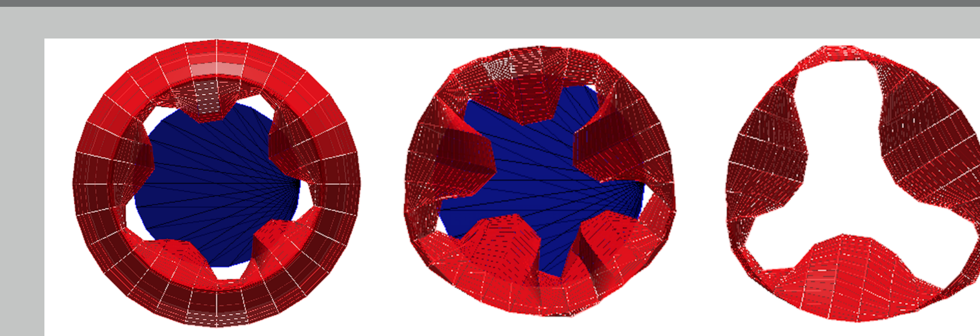

*Figure 6.* Tube buckles due to pressure drop behind rod, with circumferential wave number 5 reducing to 3, shown from end at various times during simulation.

## Discussion

As the rod enters the elastic tube, fluid (and so the tube) is pushed outward. As the pressure drops immediately behind the rod as it moves, fluid rushes inward, causing non-axisymmetric tube buckling. The buckling behavior changes with the stiffness of the springs forming the elastic, the speed of the rod, and the aspect ratio of the rod and tube.

## Future Work

- Determine causes of specific buckling behavior of passive elastic tubes.
- Use a continuum elastic model for the tube and compare system behavior.
- Increase realism with better geometry and active peristalsis in the tube.

## References

[1] C. S. Buhimschi, I. A. Buhimschi, *Advantages of vaginal delivery*, Clinical obstetrics and gynecology 49 (1) (2006) 167-183.

[2] R. Cortez, L. Fauci, A. Medovikov, *The method of regularized Stokeslets in three dimensions: analysis, validation, and application to helical swimming*, Physics of Fluids (2005).

[3] J. B. Grotberg and O. E. Jensen, *Biofluid mechanics in flexible tubes*, Annual Review of Fluid Mechanics (2004) 36:121-47.

[4] A. M. Lehn, A. Baumer, M. C. Leftwich, *An experimental approach to a simplified model of human birth*, preprint (2015).

[5] H. Nguyen and L. Fauci, *Hydrodynamics of diatom chains and semiflexible fibres*, J. R. Soc. Interface 11: 20140314 (2014).

[6] Fig.1: "HumanNewborn" by Ernest F - Own work. Licensed under CC BY-SA 3.0 via Commons - `https://commons.wikimedia.org/wiki/File:HumanNewborn.JPG#/media/File:HumanNewborn.JPG`, "Postpartum baby2" by Tom Adriaenssen - `http://www.flickr.com/photos/inferis/110652572/`. Licensed under CC BY-SA 2.0 via Commons - `https://commons.wikimedia.org/wiki/File:Postpartum_baby2.jpg#/media/File:Postpartum_baby2.jpg`

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