A numerical investigation of a simplified human birth model

Roseanna Pealatere[1], Alexa Baumer[2], Lisa Fauci[1], and Megan C. Leftwich[2]

Tulane University[1], George Washington University[2]

Work supported in part by NSF DMS 1043626

Motivation

Vaginal delivery is linked to

- shorter post-birth hospital stays
- lower likelihood of intensive care stays
- lower mortality rates [1]

Fluid mechanics greatly informs the total mechanics of birth.

- vernix caseosa
- amniotic fluid

[1] C. S. Buhimschi, I. A. Buhimschi (2006). *Advantages of vaginal delivery*, Clinical obstetrics and gynecology.

Physical Experiment

- birth canal modeled by elastic latex tube
- fetus modeled by solid glass cylinder
- amniotic fluid modeled by viscous fluid (water/methyl cellulose mixture)

The Model: Solid Behavior

Elastic Tube

- Tube modeled by network of Hookean springs.
- Force at $\mathbf{x}_l$ due to spring from $\mathbf{x}_m$:
  $\mathbf{f}(\mathbf{x}_l) = \tau \left(\frac{\|\mathbf{x}_m - \mathbf{x}_l\|}{\Delta_{lm}} - 1 \right) \frac{(\mathbf{x}_m - \mathbf{x}_l)}{\|\mathbf{x}_m - \mathbf{x}_l\|}$
- τ chosen to match elastic properties to physical experiment. [2]

Rigid Inner Rod

- A constant velocity $\mathbf{u}$ is specified in the z-direction.

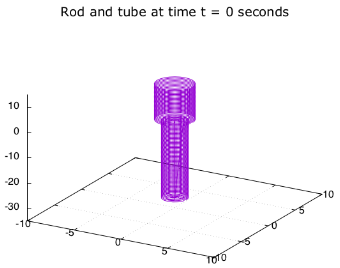


Figure : Discretization of rod and tube position in fluid at beginning of simulation.

[2] H. Nguyen and L. Fauci (2014). *Hydrodynamics of diatom chains and semiflexible fibres*, J. R. Soc. Interface.

The Model: Fluid Dynamics

Fluid Behavior is governed by the Stokes equations:

$$0 = -\nabla p + \mu \Delta \mathbf{u} + \mathbf{f},$$

$$\nabla \cdot \mathbf{u} = 0.$$

The linear relationship between fluid velocities and regularized forces localized at N points is given by

$$\mathbf{u}(\mathbf{x}) = \frac{1}{\mu} \sum_{k=1}^{K} \left[(\mathbf{f}_k \cdot \nabla) \nabla B_\varepsilon(|\mathbf{x} - \mathbf{x}_k|) - \mathbf{f}_k G_\varepsilon(|\mathbf{x} - \mathbf{x}_k|) + \mathbf{u}_b(\mathbf{x}) \right],$$
$$p(\mathbf{x}) = \sum_{k=1}^{K} \left[\mathbf{f}_k \cdot \nabla G_\varepsilon(|\mathbf{x} - \mathbf{x}_k|) \right],$$

where $\Delta B_\varepsilon = G_\varepsilon, \Delta G_\varepsilon = \phi_\varepsilon, \phi_\varepsilon(r) = \frac{15\varepsilon^4}{8\pi(r^2+\varepsilon^2)^{(7/2)}}$

Here, μ is viscosity, $\mathbf{x}_k$ are points on discretized tube and rod, $\mathbf{f}_k$ is the force at that point, and ε is a regularization parameter. [3],[4]

[3] R. Cortez (2001). *Method of Regularized Stokeslets*, SIAM Journal of Scientific Computing.
[4] R. Cortez, L. Fauci, A. Medovikov (2005). *The method of regularized Stokeslets in three dimensions: analysis, validation, and application to helical swimming*, Physics of Fluids.

The Model: Numerical Solution

Using the solution to the regularized Stokes equations for a given blob function, we can

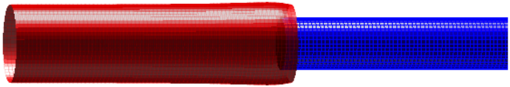
(1) find the velocity induced on the rod by spring forces in the tube,

(2) solve for any additional forces on the rod necessary to achieve its prescribed velocity,

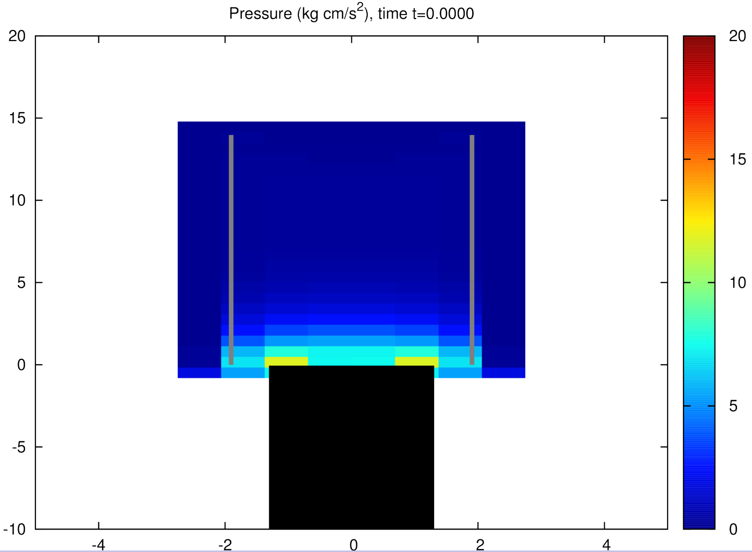
(3) evaluate the velocity and pressure at every point in the system,

(4) update the tube and rod positions using these velocities one step forward in time.

System Behavior



Fluid Pressure



Tube Buckling

Pulling Force

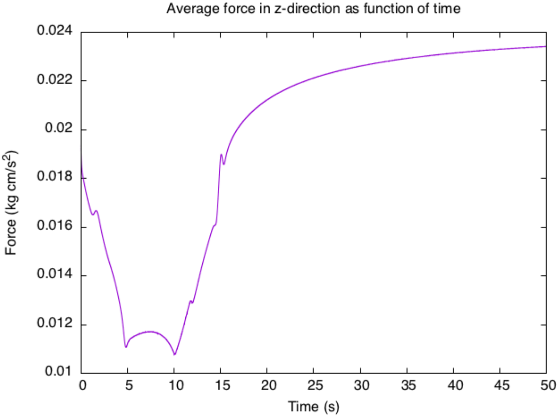
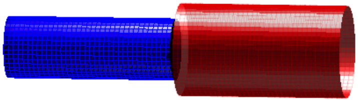
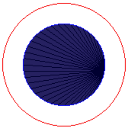


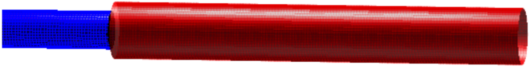
Figure : Necessary pulling force to move rod through tube at prescribed constant velocity, plotted against time.

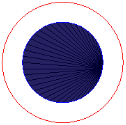
Tube Buckling





Tube Buckling





Future Work

- Continuum model of elastic tube
- Increasing realism
  - active elastic tube / modeling peristalsis
  - more accurate geometry
- Further analysis of tube buckling behavior

Slides available at
`dauns.math.tulane.edu/~rpealate`