A simplified human birth model: translation of a rigid cylinder through a passive elastic tube

Roseanna Gossmann[1]*, Alexa Baumer[2], Lisa Fauci[1], Megan C. Leftwich[2]

Tulane University[1], The George Washington University[2]

Motivation

When compared with Caesarean delivery, vaginal delivery is linked to

- shorter post-birth hospital stays
- lower likelihood of intensive care stays
- lower mortality rates [1]

Greater understanding of the causes of force on the infant during childbirth could decrease the occurrence of unnecessary Caesarean deliveries. Fluid mechanics greatly informs the total mechanics of birth. [4] We aim to discover how the involved fluids affect forces on the infant during birth.

Experimental Parameters

- Rigid acrylic cylinder (fetus) pulled through center of passive elastic tube (birth canal) at set velocity
- System immersed in methyl cellulose in water (amniotic fluid)

Physical experiment at Leftwich Laboratory[2]

- Rigid inner cylinder radius

  $$R_C = 1.27, 1.11125, 0.9525 \text{ cm}$$

- Rigid inner cylinder length

  $$L_C = 6.6, 13.2 \text{ cm}$$

- Velocity of inner cylinder

  $$U = 0.4, 0.8, 1.6, 3.2 \text{ cm/s}$$

Mathematical Background

Much work has been done studying fluid flow through elastic tubes with fixed ends in three dimensions. [3]

- In previous numerical models, tube dynamics have been modeled using nonlinear shell theory and viscous fluid dynamics using lubrication theory.
- Non-axisymmetric tube collapse occurs when the transmural pressure reaches a critically low value.

Numerical Methods

Elastic tube

- Tube modeled by network of Hookean springs.
- Force at $\mathbf{x}_l$ due to spring from $\mathbf{x}_m$:

  $$\mathbf{g}(\mathbf{x}_l) = \tau\left(\frac{\|\mathbf{x}_m - \mathbf{x}_l\|}{\Delta_{lm}} - 1\right)\frac{(\mathbf{x}_m - \mathbf{x}_l)}{\|\mathbf{x}_m - \mathbf{x}_l\|}$$

- τ chosen to match elastic properties to physical experiment. [5]

Rigid inner cylinder

- A constant velocity $\mathbf{U}$ is specified in the z-direction.

Fluid governed by the Stokes equations:

$$0 = -\nabla p + \mu\Delta\mathbf{u} + \sum_{k=1}^{N}\mathbf{f}_k,$$
$$\nabla\cdot\mathbf{u} = 0,$$

where $\mathbf{f}_k$ is the total force on the point $\mathbf{x}_k$.

The linear relationship between fluid velocity and pressure and regularized forces localized at N points is given by

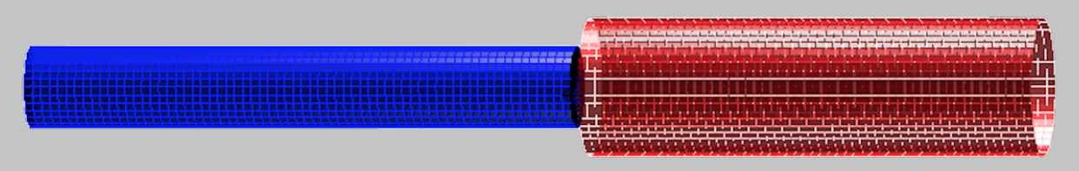
$$\mathbf{u}(\mathbf{x}) = \frac{1}{\mu}\sum_{k=1}^{N}\left[(\mathbf{f}_k\cdot\nabla)\nabla B_\varepsilon(|\mathbf{x}-\mathbf{x}_k|) - \mathbf{f}_k G_\varepsilon(|\mathbf{x}-\mathbf{x}_k|) + \mathbf{u}_b(\mathbf{x})\right],$$

$$p(\mathbf{x}) = \sum_{k=1}^{N}\left[\mathbf{f}_k\cdot\nabla G_\varepsilon(|\mathbf{x}-\mathbf{x}_k|)\right],$$

where $\Delta B_\varepsilon = G_\varepsilon, \Delta G_\varepsilon = \phi_\varepsilon = \frac{15\varepsilon^4}{8\pi(r^2+\varepsilon^2)^{(7/2)}}$, μ =viscosity, ε regularization parameter. [2]

Algorithm

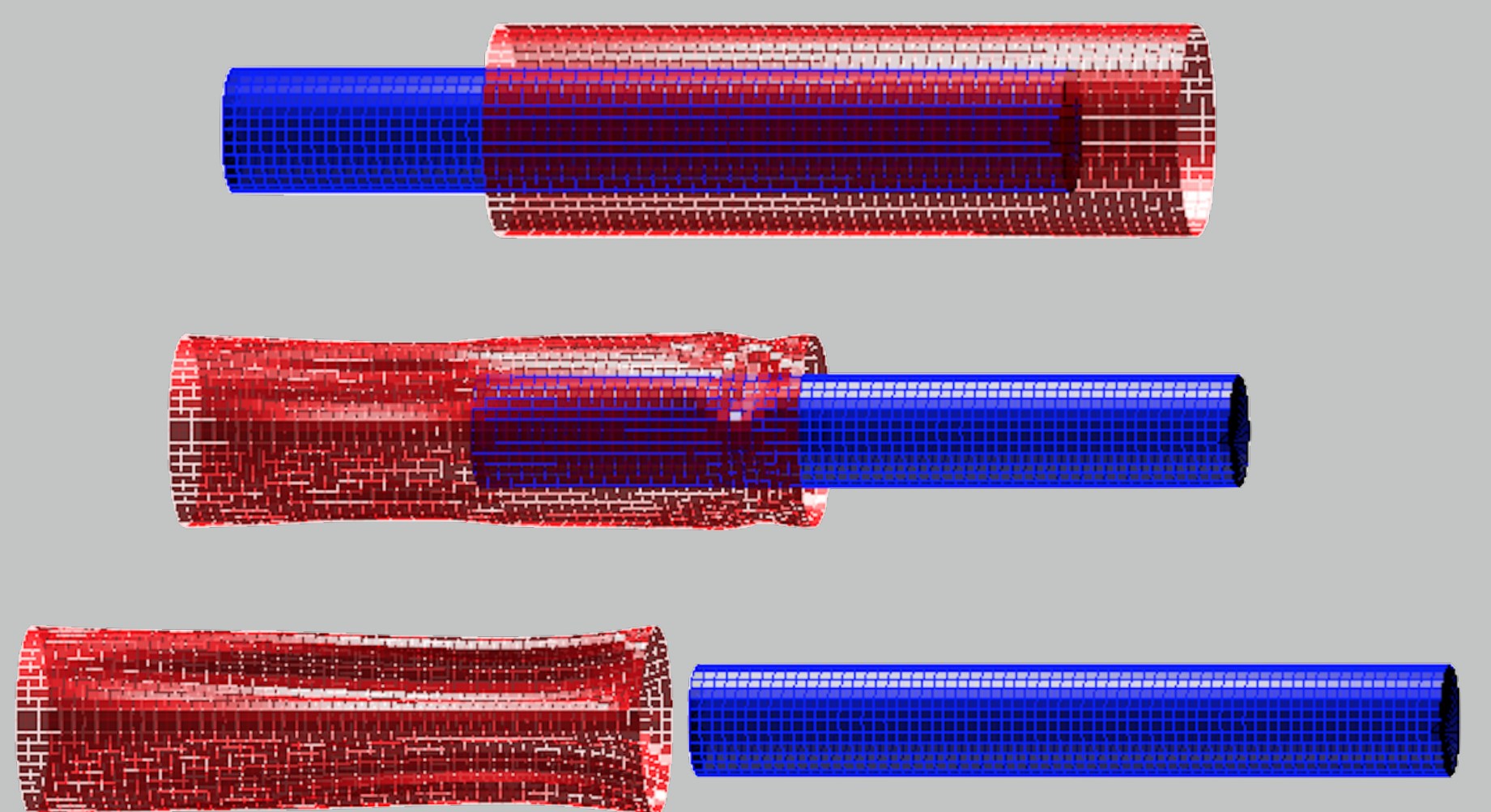
(1) find velocity induced on the rod by spring forces,
(2) solve for additional forces necessary for prescribed velocity,
(3) evaluate velocity and pressure throughout system,
(4) update tube and cylinder positions one time-step,
(5) repeat.

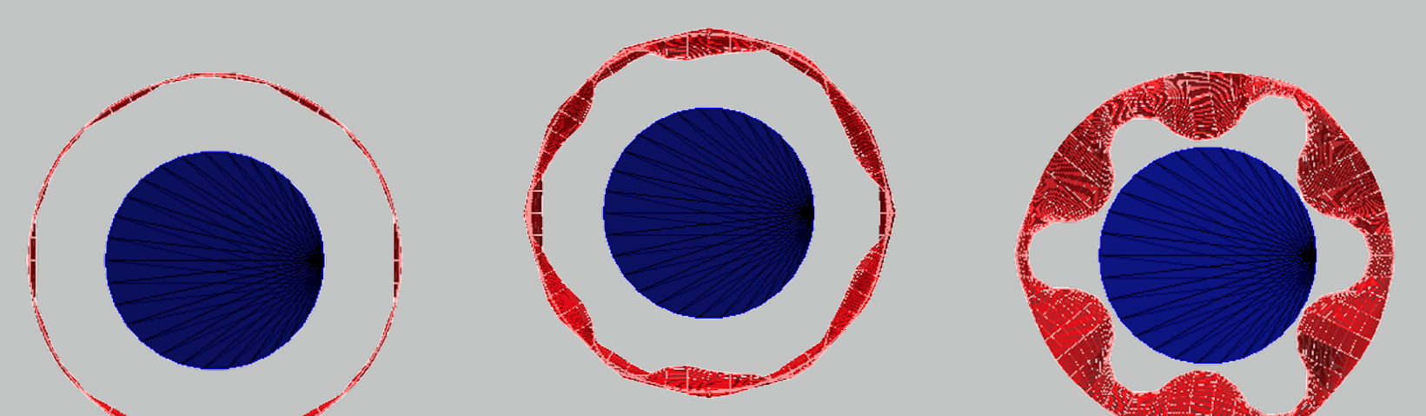


Inner cylinder (blue) and elastic tube (red) at start of simulation for $R_C = 0.9525, L_C = 13.2$.

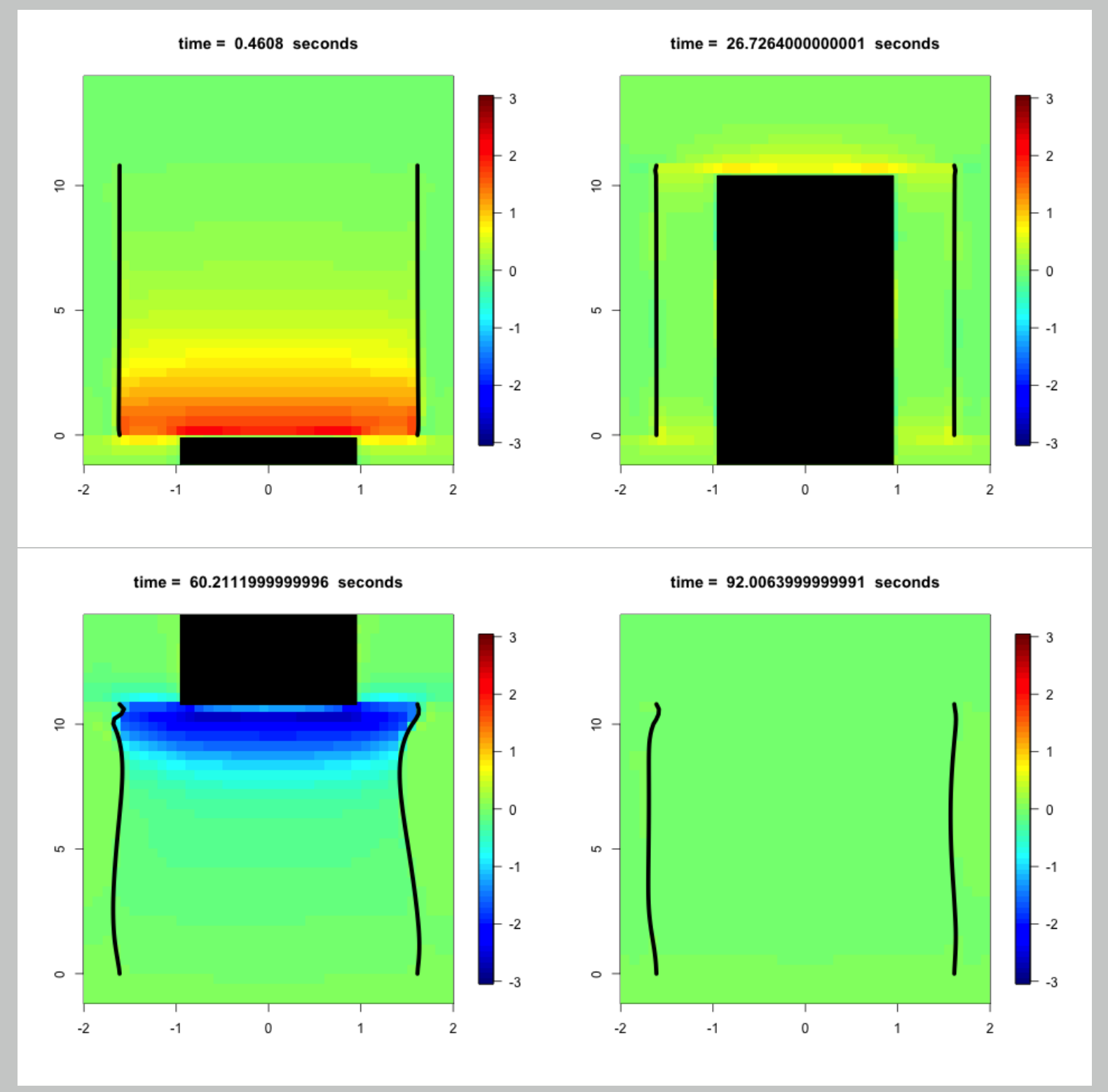
Simulation Results

$R_C = 0.9525, L_C = 13.2, U = 0.4$ cm/s: As the rigid inner cylinder moves through the elastic tube, with tube ends remaining fixed in space, the tube buckles behind the trailing end of the cylinder as the fluid pressure drops.

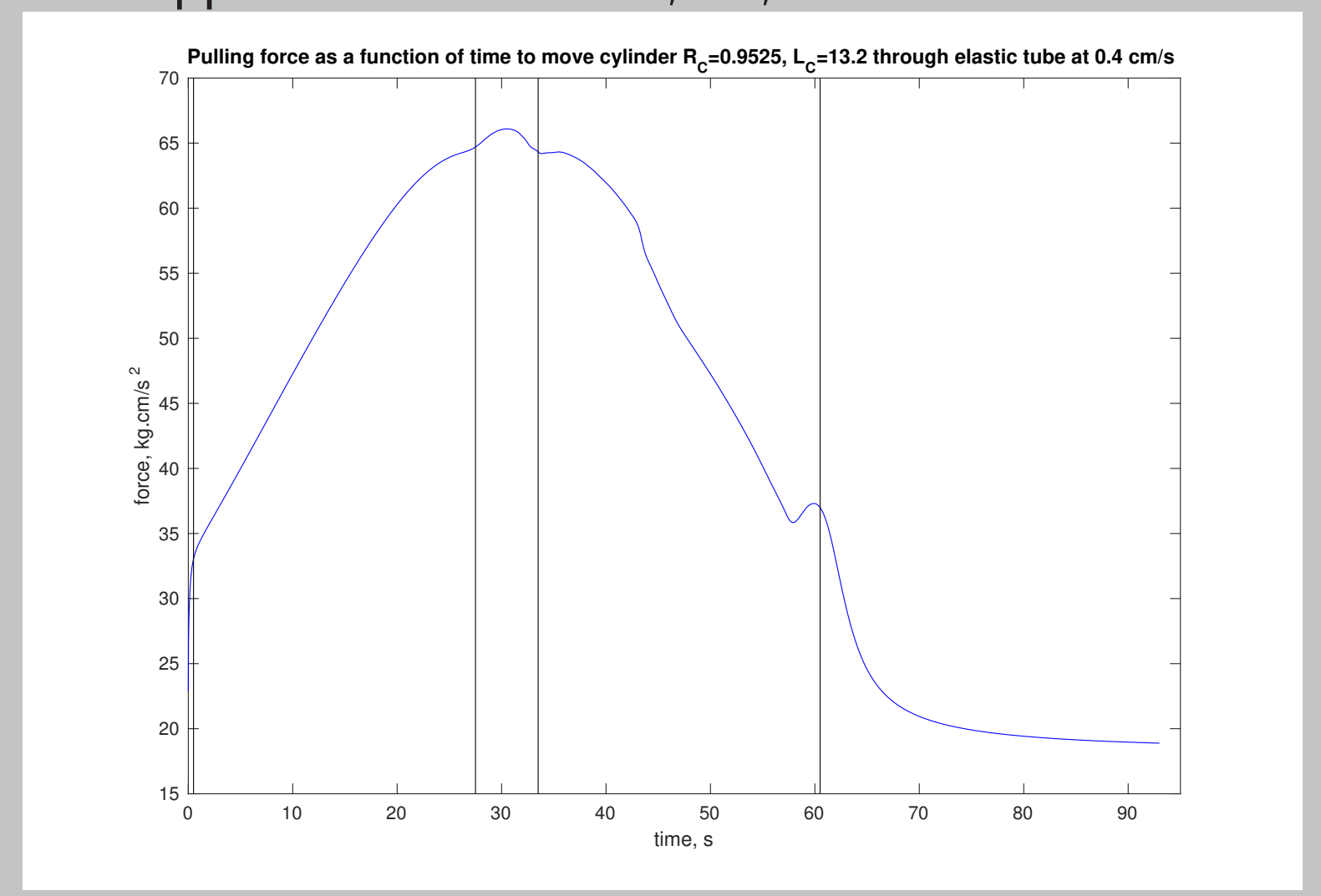




Tube and inner cylinder seen from trailing end as six-fold buckling develops due to pressure drop; positions shown to left, approximate times 22, 45, and 62 seconds.

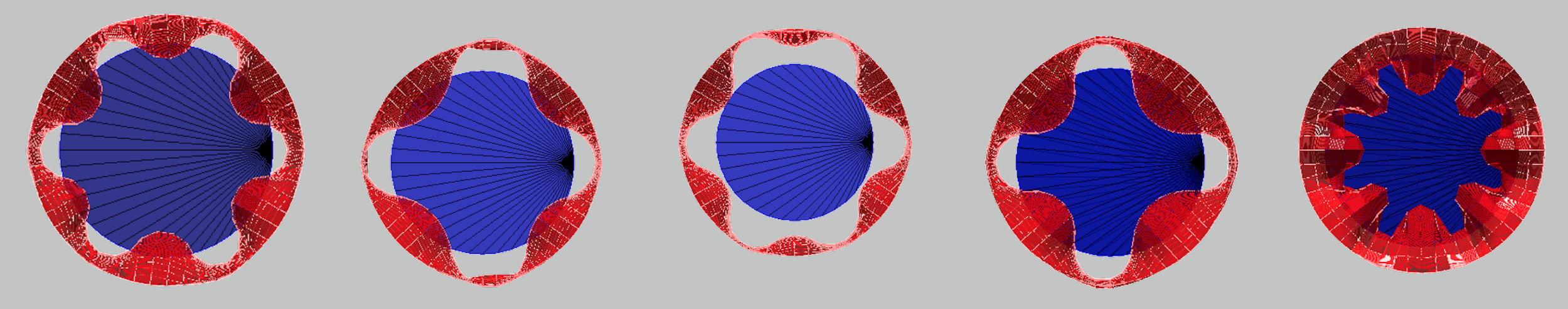


Tube, rod, and fluid pressure in cross section.



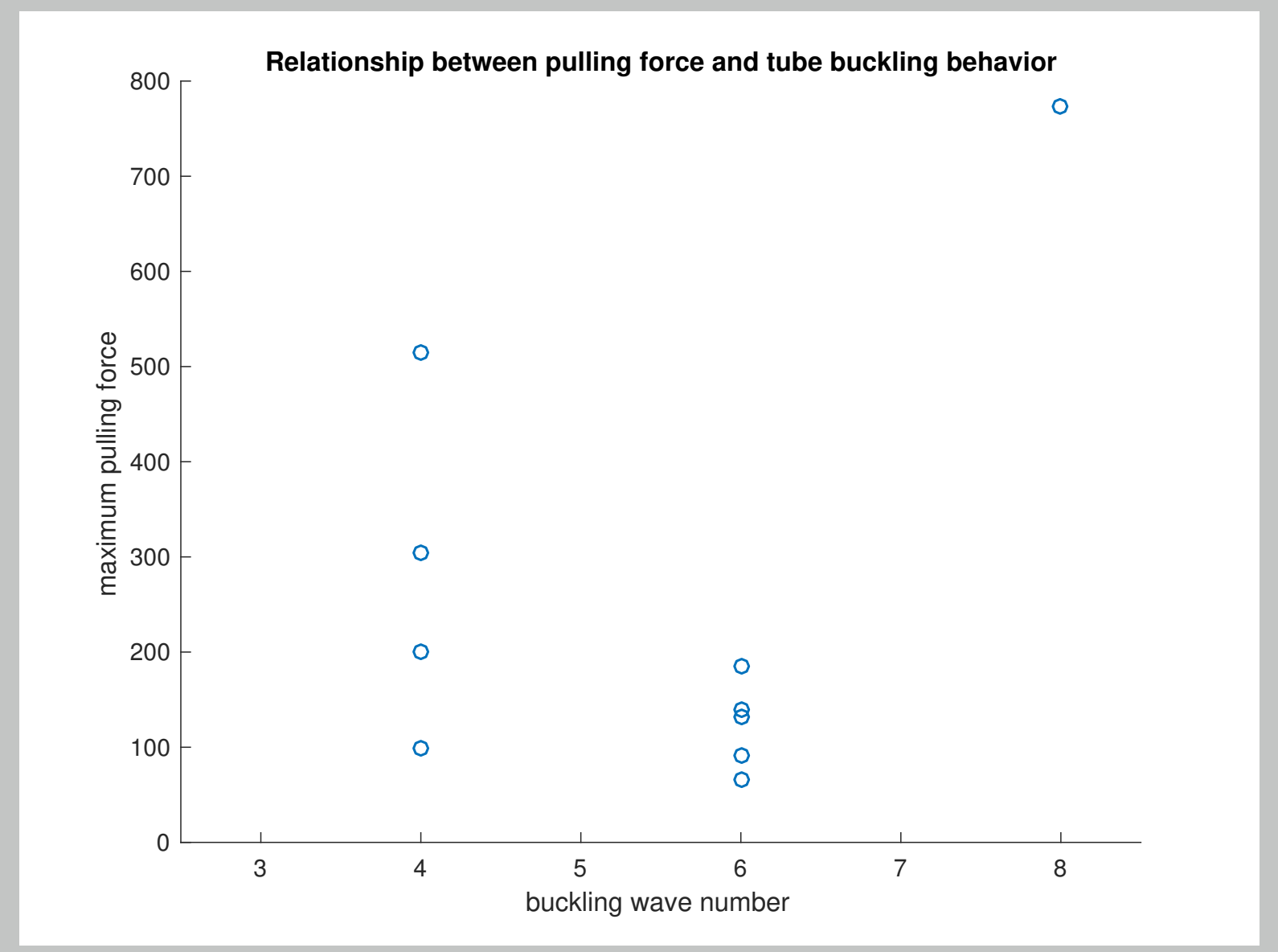
Pulling force necessary to keep rigid inner cylinder moving through tube at constant velocity 0.4 cm/s, shown as a function of time. Black lines delineate times at which the inner cylinder enters the elastic tube, is completely inside the tube, begins exiting the tube, and is completely outside the tube. Force rises as the cylinder enters and drops as it exits, with one small peak right before it exits the tube fully, probably due to increased cylinder-tube interaction because of tube buckling.

Tube Buckling Variation



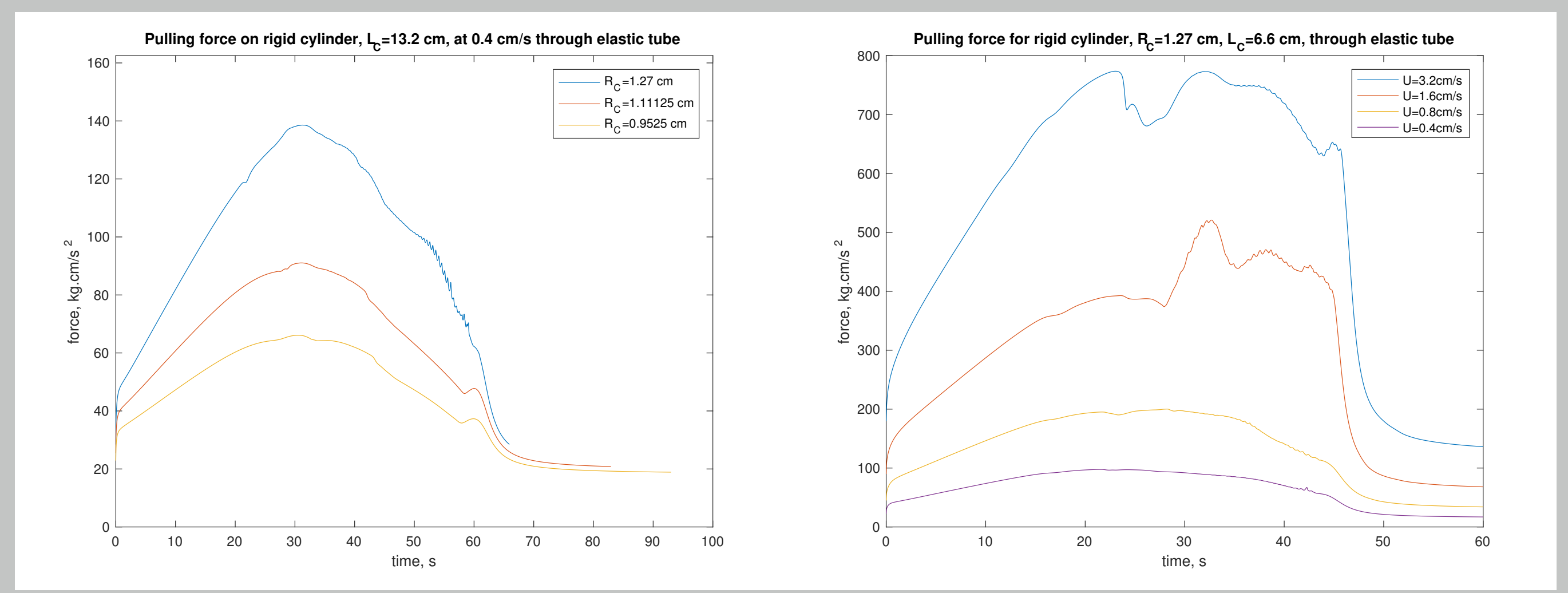
A range of buckling behavior exhibited for differing cylinder geometry and velocity.

Pulling Force and Tube Buckling



The relationship between maximum pulling force and buckling behavior is considered. Four-fold buckling occurred for forces ranging from 100 to 515 kg·cm/s^2, six-fold buckling for forces 50 to 200, and eight-fold buckling for a force of 780.

Force and Velocity



Greater force is necessary to move cylinders of greater width at the same velocity through the tube (left). Force approximately doubles as velocity doubles for the same cylinder geometry, as expected due to linearity of the Stokes equations (right).

Future Work

- Determine causal relationship between force and other variables and specific buckling behavior of the elastic tube.
- Use a continuum elastic model for the tube and compare system behavior; consider nonzero Reynolds numbers.
- Increase realism with better geometry and active peristalsis in the tube.

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*Contact: rpealate@tulane.edu